1) Use Gauss divergence Theorem carefully to find the outward flux $\iint_S F.n \, d\sigma$

of the vector field $F = 1/\rho < 9x$, y, $8\rho^{-2} z >$ where $\rho = \sqrt{x^2 + y^2 + z^2}$ across i) the surface $\rho = 2$ ii) the surface surrounding the region $1 \le \rho \le 2$ between two spheres. 2) Use <u>Stokes' Theorem</u> to find the outward flux of CURL(F)

 $\iint_{S} \operatorname{Curl}(F).n \, d\sigma \text{ of the vector field } \mathbf{F} = \langle 6\mathbf{y}, 4\mathbf{x}, \mathbf{z} \rangle \text{ across the upper part of the ellipsoid } \mathbf{x}^{2} + 2\mathbf{y}^{2} + \mathbf{z}^{2} = 16 \text{ whose boundary C lies on the plane } \mathbf{z} = \mathbf{x} + 4.$ Hints: C also lies on the cylinder $(\mathbf{x} + 2)^{2} + \mathbf{y}^{2} = 4.$ dz = dx (but why?) **3a)** Let $\mathbf{F} = \langle 5\mathbf{y}, 8\mathbf{x}, \mathbf{z} \rangle$ and let S be the <u>open</u> paraboloid $z = 2(x^2 + y^2) - 8 \& z \leq 0$. Find $\iint_S F.n \, d\sigma$ (where n is the outer normal vector to our surface) by Gauss Divergence Theorem (by using a standard trick) **3b)** Let $\mathbf{F} = \langle 5\mathbf{y}, 8\mathbf{x}, \mathbf{z} \rangle$ and let S be the <u>open</u> paraboloid $z = 2(x^2 + y^2) - 8 \& z \le 0$. Find $\iint_{S} \operatorname{Curl}(F) \cdot n \, d\sigma$ (by any method) (where n is the outer normal vector to our surface) Reminder: The boundary C is on z=0. 4a) Change the following D.E to a linear DE in standard form then STOP!

$$\mathbf{x}\mathbf{y}' - (\mathbf{x}^2 + 1)\mathbf{y}^{5/4} = (\frac{\mathbf{x}^2}{\mathbf{x}^4 + 1})\mathbf{y}^{5/4}$$

4b) Change the following (non-exact) D.E <u>to exact then STOP</u>! $(8-7\mathbf{y}+\mathbf{x}^3\mathbf{e}^x) \mathbf{dx} + \mathbf{x}\mathbf{dy} = 0$ **5a)** Simplify $\nabla . (\nabla f)$ and $\nabla . (\nabla f + f \nabla g)$ in terms of f & g and their partial derivatives.

5b) Set up the integrals (but do not evaluate) to find the surface area of the ellipsoid $4x^2 + 9y^2 + z^2 = 16$

(Hint: You may use symmetry (with $\mathbf{z} \ge 0$)

6a) Change the DE $(7y^2 + 2xy)y' = 3x^2 + 4xy \sin(\frac{7x + y}{x + 2y})$ to a separable DE. **Then STOP!!**

6b) Change (by substitution) the DE $xy' = y \sin(xy)$ into a separable DE. <u>Then STOP!</u>

7a) If $\{y'=f(x,y) ; y(1)=1\}$ has 3 solutions, does this contradict our Existence/uniqueness Theorem for IVP?

7b) If $\{y'=f(x,y) ; y(1)=1\}$ has no solutions, does this contradict our Existence/uniqueness Theorem for IVP?

7c) If { y'=f(x,y) ; y '(1)=1 } has no solutions, does this contradict our Existence/uniqueness Theorem for IVP ?

7d) Find all 3 solutions of the IVP: $y' = y^{1/3}$; y(1) = 0.